# ECE 162 Week 5 – Thévenin and Norton Equivalent Circuits

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## Purpose

The purpose of this lab is to analyze a complex circuit using Norton and Thévenin equivalent circuits.

## Theory

In this lab we experiment with Norton and Thévenin equivalent resistances, and compare the values found experimentally with values found theoretically.

Thévenin’s theory says that it’s possible to simplify linear circuits to an equivalent circuit with only one voltage source and one series resistance connected to a load resistance. This is especially useful for analyzing power across a specified load resistor. This load resistor (RL) is often subject to change, and reanalysis of the circuit for each iteration of load resistance can be tedious. Creating an equivalent Thévenin circuit makes this process easier. For the circuit in question, all values are linear; therefore Thévenin’s theory can be applied.

Thévenin’s theory works by first removing the load resistor temporarily, and by reducing the remaining aspects of the circuit to a single voltage source and series resistor. The load resistance is then connected to the Thévenin equivalent circuit and calculations can be carried out as if the whole circuit is a simple series circuit. This Thévenin equivalent circuit is shown below in Figure 1:

Figure 1

The advantage of creating the equivalent Thévenin circuit, of course is the simplicity of calculating properties of the load resistor (such as power in this example).

The voltage across the load resistor is called the Thévenin voltage. This is can be calculated by using the current and known resistance values to find the voltage dissipated by the resistor.

Next the load resistor is “taken out” of the system by replacing it with a short circuit. The current flowing across this short circuit where the load resistor is then found and this value is known as the equivalent Norton current (IN). This can be done in a variety of ways, but in this experiment it was found using Kirchhoff voltage law (shown in results section).

Finally we can calculate the Thévenin resistance. This is done using the Thévenin voltage and equivalent Norton current. The equation Thévenin resistance is shown below:

Filling in the values for RTh and VTh in the equivalent Thévenin circuit above in Figure 1 allows for simplified calculation of power for the load resistor. This is helpful, as there are 3 different power calculations that are needed for this lab, and it would become increasingly tedious to calculate these values as the number of iterations increased.

## Experimental Method

* Create the circuit shown in figure 1.33 in the textbook. Replace each of the 2 Ω resistors with a 2.2 kΩ resistors (note: this is different than what was originally asked, but that was the experimental method for our group). Replace the remaining resistors with 1 kΩ resistors.
* Measure VTh across the load resistor by placing both ends of a voltmeter on either end of the resistor.
* Measure IN across the load resistor while connected to the external circuit and the 10 V source.
* Measure RTh across the load resistor.
* Connect a load resistor roughly equal to RTh in parallel to the load resistor (between terminals a and b in Figure 1). Measure the power dissipated. Iterate twice, once for a resistor of roughly twice the resistance, and once for a resistor of roughly half the resistance.
* Compare the measured values with those calculated in the results section

## Diagram

Shown below is the circuit diagram for this lab (Figure 2):

Figure 2

## Results

For the circuit diagram shown above in Figure 2, the currents were calculated for every resistor. This was done by doing a Kirchhoff current law for three different pathways. This produced the following series of equations (2-4):

This produces the following result:

By looking at the circuit diagram and by utilizing Ohm’s Law, we can calculate the value of the Thévenin voltage. This is shown below in equation 5:

Now that we have the equivalent Thévenin voltage, we can move on to finding the Norton current. This is done by replacing the resistor with a short circuit and measuring the current going through the new short circuit. Below is a figure showing the circuit diagram with the load resistor as a short circuit (Figure 3):

We calculate the current going across the short circuit in the same way we calculated through the load resistor, with Kirchhoff voltage law. The equations for the system without the load resistor are slightly different than with, so those equations are shown below as well (equations 6-8). These currents are differentiated from the previous by denoting the subscript N (for Norton).

By making the load resistor a short circuit, the equations become a bit easier, and we only have to solve a system of two equations and two unknowns. The solutions to these equations are shown below.

Using this, we are able to calculate the Norton equivalent current by finding the current through the short circuit. The equation used is as follows (equation 9):

The Thévenin resistance can now be found using equation 1. The result of this calculation is that the Thévenin resistance is 488.6 Ω.

Using the Thévenin resistance and equivalent voltage, it is much easier to calculate different iterations of the power dissipated by the selected resistors. Since the load resistor is placed in series with the Thévenin resistance, the current is found using Ohm’s Law. Using this value for each resistor, power can be calculated using equation 10 below:

Three iterations were made for calculating and measuring power. It was recommended that we use one resistor close to the Thévenin resistance, so we decided to use a 470 Ω resistor. It was also recommended to use a resistor with half the Thévenin resistance, so we decided to use a 220 Ω resistor for this iteration. Finally it was recommended we use a resistor approximately double the value of the Thévenin resistance, so we used a 1 kΩ resistor.

Below is a table describing all the values, measured and calculated, including the percent error of each value.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| VTh (V) | IN (mA) | RTh (Ω) | P(RTh) (W) | P(2RTh) (W) | P(0.5RTh) (W) |
| Experimental Values | | | | | |
| 5.762 | 11.6 | 485.8 | .0227 | .0141 | .0107 |
| Calculated Values | | | | | |
| 5.79 | 11.85 | 488.6 | .0171 | .0151 | .0147 |
| 0.48% | 2.11% | 0.57% | 24.66% | 6.62% | 27.2% |

Table 1

## Discussion

For the first three values calculated (VTh, IN, RTh), the percent error was negligible. These values were relatively easy to measure, did not have too many sources of variation, and the tools used to measure these values were very precise. This all led to a low percent error.

On the other hand, the power values were more difficult to get a low percent error. This is in part due to the compounding of several factors. For one, we were approximating the circuit, including all of the resistors and the power source into one equivalent circuit, and placing that circuit in series with the load resistor. There were many different connecting wires which were not taken into account, and this would affect the equivalent circuit. Another thing that could affect the percent error is the fact that we were choosing resistors that were “close to” the value of the Thévenin resistance. This will skew the calculations, and again increase the percent error.

It is clear that there are more aspects which could affect the percent error of the power calculations, and that is why those values have generally a greater percent error than the preceding three.

## Conclusion

This lab shows the power of creating Norton and Thévenin resistance circuits. To be able to iterate through power calculations with any other method would be incredibly tedious, but with Norton and Thévenin equivalent circuits it turns even the most complex circuit systems into a system with just two resistors in series and a voltage source. Changing load resistance is very common in power systems, as multiple loads may be switched on and off as needed. Norton and Thévenin resistances provide a nifty way to account for this.